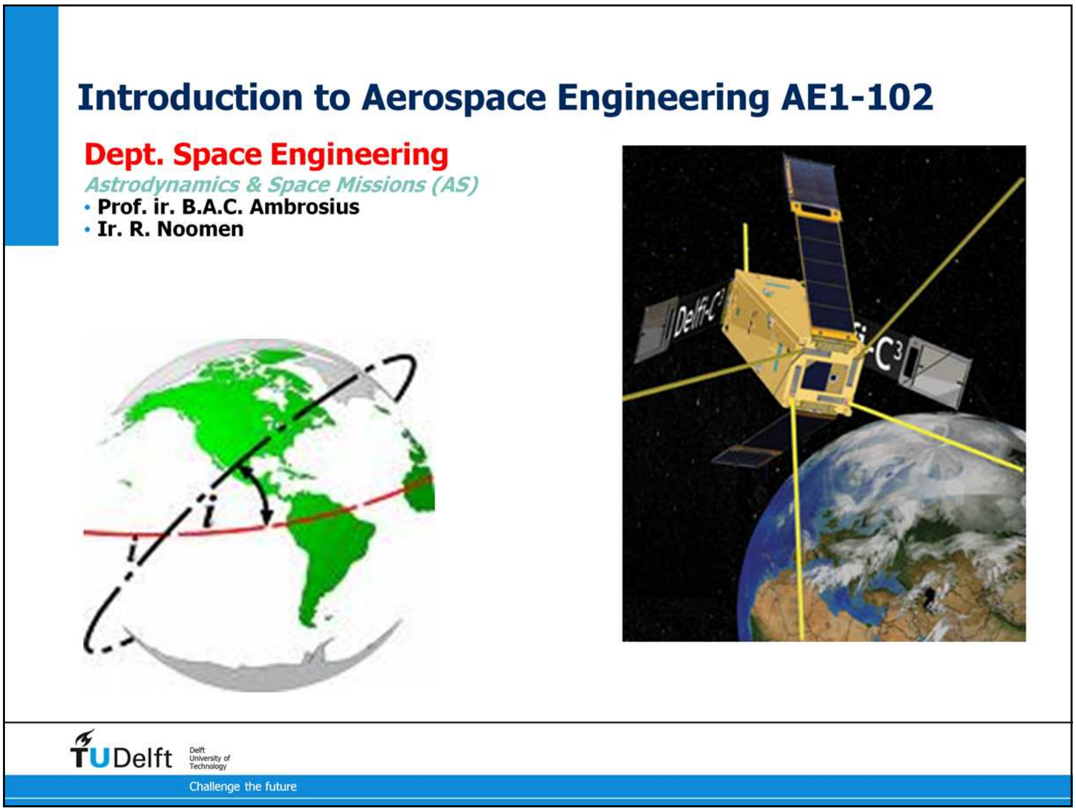
Introduction to Aerospace Engineering

Lecture slides



Part of the lecture material for this chapter originates from B.A.C. Ambrosius, R.J. Hamann, R. Scharroo, P.N.A.M. Visser and K.F. Wakker.

References to “”Introduction to Flight” by J.D. Anderson will be given in footnotes where relevant.

7 - 8

Orbital mechanics: satellite orbits (2)

November 17, 2009

This topic is (to a large extent) covered by Chapter 8 of “Introduction to Flight” by Anderson.

General remarks

Two aspects are important to note when working with Anderson's "Introduction to Flight" and these lecture notes:

- The derivations in these sheets are done per unit of mass, whereas in the text book (p. 603 and further) this is not the case.
- Some parameter conventions are different (see table below).

parameter	notation in "Introduction to Flight"	customary notation
gravitational parameter [m^3/s^2]	k^2	GM, or μ
constant for angular momentum	h	H

Learning goals

The student should be able to:

- classify satellite maneuvers and quantify the required propellant usage
- describe and explain the principle of Hohmann transfer orbits
- compute relevant parameters (e.g. ΔV, flight time) for a Hohmann transfer
- generate a realistic ΔV budget for an arbitrary space mission

Lecture material:

- these slides (incl. footnotes)

Anderson's "Introduction to Flight" (at least the chapters on orbital mechanics) is NOT part of the material to be studied for the exam; it is "just" reference material, for further reading.

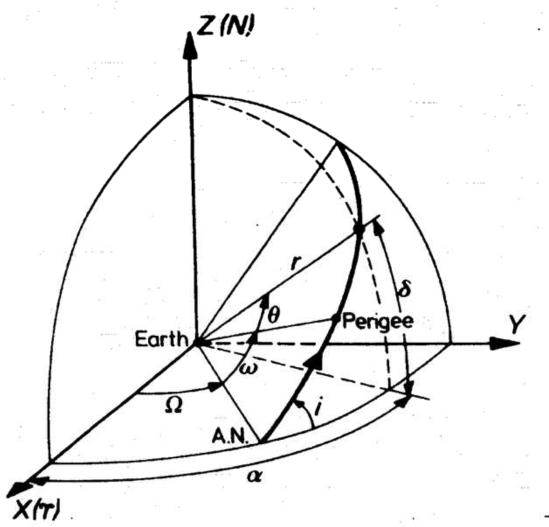
Questions

- What is the change in velocity that I need to rendez-vous with the International Space Station?
- Idem, to change the inclination of my orbit from 29.8° to 0°
- How can I evaluate different strategies to end up in a GEO position?
- How much ΔV is needed to travel to other celestial bodies?
- What is the minimum amount of ΔV to travel to Mars? To Neptune?

Fundamentals (the 3rd dimension)

3-dimensional orbits: another 3 Kepler elements:

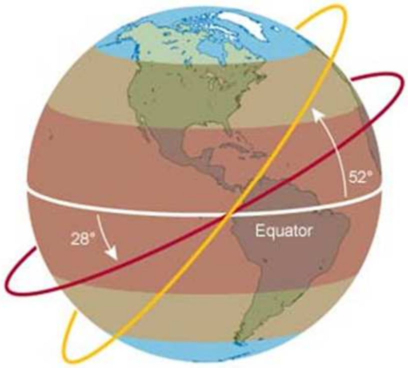
- i – inclination [deg]
- Ω - right ascension of ascending node [deg]
- ω – argument of pericenter [deg]



The inclination “i” is the angle between the orbital plane and a reference plane, such as the equatorial plane. It is measured at the ascending node, i.e. the location where the satellite transits from the Southern Hemisphere to the Northern Hemisphere, so by definition its value is between 0° and 180°. The parameters Ω and ω can take any value between 0° and 360°.

Overview

- maneuvers
- Hohmann orbits
- deltaV budgets



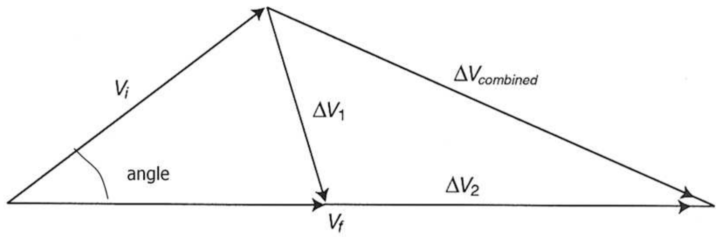
[Skyandtelescope, 2009]

Maneuvers

purpose:
- Orbit transfers (e.g. LEO → GTO → GEO)
- Plane changes (e.g. launch from Kourou, inc = 5°, to equatorial, inc = 0°)
- Rendez-vous
- Orbit maintenance

Question:
- Where optimum (i.e. minimum propellant mass)?

Maneuvers (cnt'd)



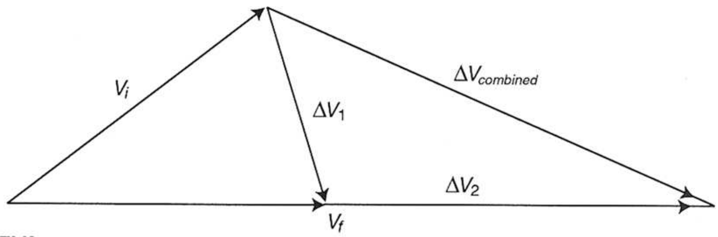
General:

- Velocity before maneuver $\mathbf{V_i}$
- Velocity after maneuver $\mathbf{V_f} = \mathbf{V_i} + \mathbf{\Delta V_{combined}}$
- Energy gain $\Delta E = ½ V_f^2 – ½ V_i^2 = ½ (\Delta V)^2 + \mathbf{V_i} \cdot \mathbf{\Delta V_{combined}}$

Best strategy?

This is an illustration of a most general maneuver: both in-plane and cross-plane. Important: we assume that maneuvers take effect instantaneously, i.e. a ΔV is achieved in an infinitesimal small time-step: the so-called impulsive shot.

Maneuvers (cnt'd)



Best strategy:

- general: tangential to "old" velocity;
- in-plane maneuvers: where V_i is largest (*i.e.* in pericenter);
  $\Delta V = |V_f - V_i|$
- out-of-plane (or: dog-leg): where V_i is smallest (*i.e.* in apocenter);
  $\Delta V = 2\ V \sin(\Delta i/2)$
- Both in-plane and out-of-plane: combine in one shot ($\Delta V_{combined} \leq \Delta V_1 + \Delta V_2$); cosine rule:
  $\Delta V_{combined}^2 = \Delta V_i^2 + \Delta V_f^2 - 2\ \Delta V_i\ \Delta V_f \cos(\Delta i)$

Here, Δi is the angle between V_i and V_f. The best strategies for the first 3 cases are based on the equation on the previous page, whereas the 4th conclusion follows from elementary mathematics for triangles.

Maneuvers (cnt'd)

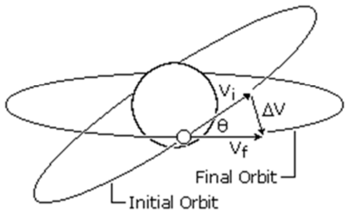
Example 1:
Increase velocity in circular parking orbit at 185 km to parabolic velocity.

Answer:
- Velocity in circular parking orbit = $\sqrt{(\mu/r)}$ = 7.793 km/s
- Velocity in pericenter parabolic orbit = $\sqrt{(2\mu/r)}$ = 11.021 km/s
- $\Delta V = V_{para} - V_{circ}$ = 3.228 km/s

μ_{Earth} = 398600.44 km^3/s^2; R_{Earth} = 6378.137 km

Maneuvers (cnt'd)

Example 2:
Change inclination of circular parking orbit at 185 km from 29.8° to 0°.



[Braeunig, 2009]

Answer:

- Velocity in circular parking orbit at 29.8° = $\sqrt{(\mu/r)}$ = 7.793 km/s
- Velocity in circular parking orbit at 0° = 7.793 km/s
- $\Delta V = 2\,V \sin(\Delta i/2)$ = 4.008 km/s

μ_{Earth} = 398600.44 km^3/s^2; R_{Earth} = 6378.137 km

$\sin(\Delta i/2) = (\Delta V/2)/V$.

Maneuvers (cnt'd)

Questions:

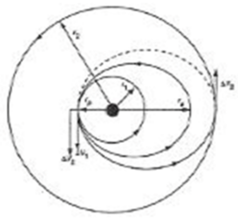
1. How much ΔV would be needed to transfer from a circular parking orbit at 185 km altitude to the pericenter of an orbit with h_p = 185 km and h_a = 35822 km? The orbits are coplanar, i.e. within the same plane.
2. How much ΔV would be needed to change the inclination of an orbit from 29.8° to 0°, if both orbits are circular and at an altitude of 35822 km?
3. How much ΔV would be needed to transfer from a circular parking orbit at 185 km altitude and with an inclination of 29.8°, to the pericenter of an orbit with h_p = 185 km and h_a = 35822 km, but with an inclination of 0° (i.e. in a single, combined maneuver)?

ANSWERS: SEE FOOTNOTES BELOW (BUT TRY FIRST!!!!)

Answers (**DID YOU TRY FIRST?**):

1. ΔV = 2.460 km/s
2. ΔV = 1.581 km/s
3. ΔV = 5.214 km/s

Maneuvers (cnt'd)



Questions:

4. How much ΔV would be needed to transfer from a circular parking orbit at 185 km altitude to the pericenter of an orbit with h_p = 185 km and h_a = 35822 km? The orbits are coplanar, i.e. within the same plane.
5. Idem, from a circular orbit at 185 km to an orbit with h_p = 185 km and h_a = 1000 km?
6. Idem, from an orbit with h_p = 185 km and h_a = 1000 km to an orbit with h_p = 185 km and h_a = 20000 km?
7. Idem, from an orbit with h_p = 185 km and h_a = 20000 km to an orbit with h_p = 185 km and h_a = 35822 km?
8. Add the ΔVs from questions 5-7, and compare with the result of question 4. Discuss

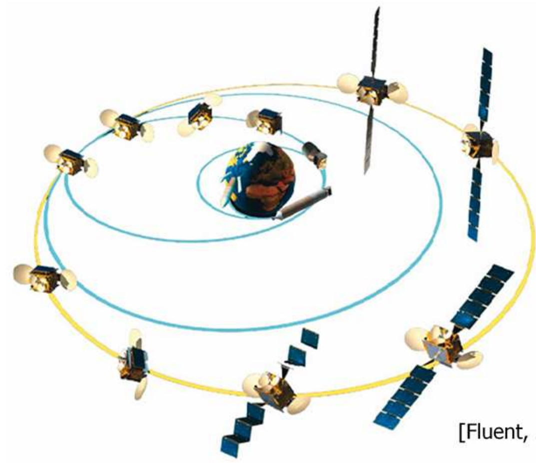
ANSWERS: SEE FOOTNOTES BELOW (BUT TRY FIRST!!!!)

Answers (**DID YOU TRY FIRST?**):

4. ΔV = 2.460 km/s
5. ΔV = 0.225 km/s
6. ΔV = 1.845 km/s
7. ΔV = 0.390 km/s
8. identical. Interpretation?

Maneuvers (cnt’d)

Another option for a stepped approach to reach GEO:



[Fluent, 2009]

Question: why would we consider to do this in stepped approach?

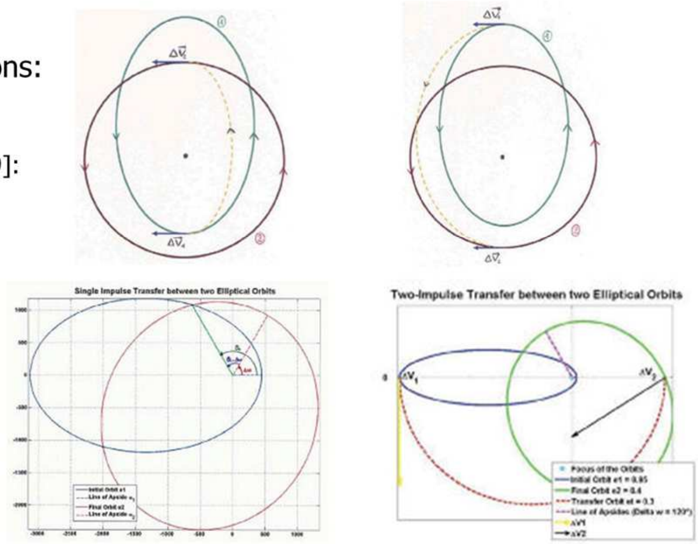
Notice the order of activities: (1) apocenter-raising maneuver, (2) series of pericenter-raising maneuvers, and (3) the deployment of the solar panels.

In-plane transfer: 2 orbits

some options:

[Locoche, 2009]:

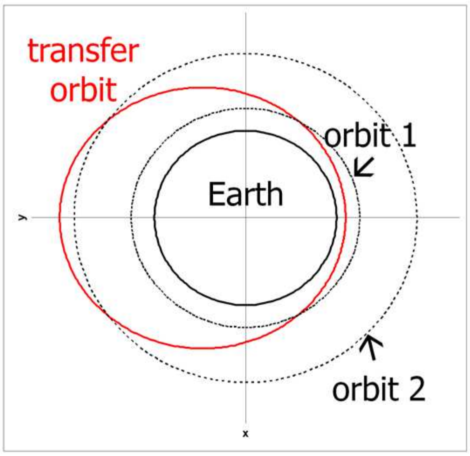
what's best?



In principle, all options are possible. But: what is efficient in energy?

In-plane transfer: LEO → GEO

- 2 impulsive shots: (1) at begin of transfer, and (2) at end of transfer
- Transfer orbit must intersect initial orbit and target orbit
- Energy needed for transfer orbit is depending on semi-major axis
- Energy efficient transfer: tangential to initial and target orbit



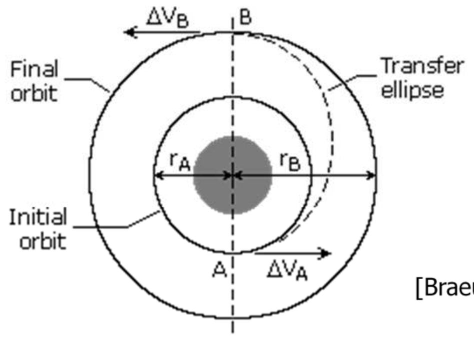
In this arbitrary case, the transfer orbit intersects the initial orbit (orbit 1) and the target orbit (orbit 2).

In-plane transfer: LEO → GEO (cnt'd)

Solution: Hohmann transfer orbit

- 2 impulsive shots: (1) at begin of transfer, and (2) at end of transfer ✓
- Transfer orbit must intersect initial orbit and target orbit ✓
- Energy needed for transfer orbit is depending on semi-major axis — Minimize semi-major axis
- Energy efficient transfer: tangential to initial and target orbit ✓

In-plane transfer: LEO → GEO (cnt'd)



[Braeunig, 2009]

- ΔV_A increases the velocity from circular (in initial orbit) to pericentric (in transfer orbit)
- ΔV_B increases the velocity from apocentric (in transfer orbit) to circular (in final orbit)

In-plane transfer: LEO → GEO (cnt'd)

circular velocity in initial orbit:

$$V_{init,c} = \sqrt{\frac{\mu}{r_{init}}}$$

eccentricity of transfer orbit:

$$e_{transfer} = \frac{r_{final} - r_{init}}{r_{final} + r_{init}}$$

semi-major axis of transfer orbit:

$$a_{transfer} = (r_{init} + r_{final})/2$$

velocity in pericenter of transfer orbit:

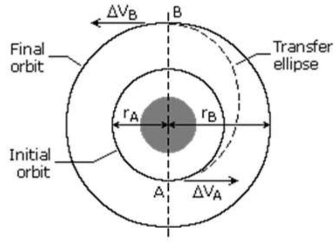
$$V_{transfer,p}^{2} = \mu\left(\frac{2}{r_{init}} - \frac{1}{a_{transfer}}\right)$$

velocity in apocenter of transfer orbit:

$$V_{transfer,a}^{2} = \mu\left(\frac{2}{r_{final}} - \frac{1}{a_{transfer}}\right)$$

circular velocity in final orbit:

$$V_{final,c} = \sqrt{\frac{\mu}{r_{final}}}$$



[Braeunig, 2009]

maneuvers:

$$\Delta V_A = V_{transfer,p} - V_{init,c}$$

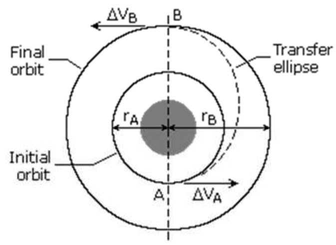
$$\Delta V_B = V_{final,c} - V_{transfer,a}$$

$$\Delta V_{total} = \Delta V_A + \Delta V_B$$

In-plane transfer: LEO → GEO (cnt'd)

Example: transfer from LEO at 185 km to GEO:

- r_{init} = 6563.137 km
- r_{final} = 42164.14 km
- $V_{init,c}$ = 7.793 km/s
- $e_{transfer}$ = 0.7306
- $a_{transfer}$ = 24363.639 km
- $V_{transfer,p}$ = 10.252 km/s
- $V_{transfer,a}$ = 1.596 km/s
- $V_{final,c}$ = 3.324 km/s
- ΔV_A = 2.459 km/s
- ΔV_B = 1.728 km/s
- ΔV_{total} = 4.187 km/s



[Braeunig, 2009]

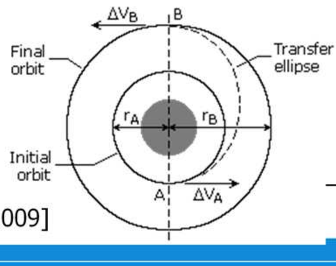
VERIFY NUMBERS!

μ_{Earth} = 398600.44 km^3/s^2; R_{Earth} = 6378.137 km

In-plane transfer: various LEO cases

Questions:

1. Compute the total ΔV needed to go from a circular parking orbit at an altitude of 185 km to the International Space Station, in a circular orbit at 350 km.
2. Compute the total ΔV needed to go from a circular parking orbit at an altitude of 350 km to a circular target orbit at 1300 km.
3. Compute the total ΔV needed to go from a circular parking orbit at an altitude of 185 km to a circular target orbit at 1300 km.
4. Compare the results of the previous 3 questions.
5. End-of-life disposal: compute the total ΔV needed to go from an circular operational orbit at an altitude of 780 km to an elliptical orbit with pericenter at 200 km and apocenter at 780 km.



Answers:

see footnotes below **(BUT TRY FIRST!!)**

[Braeunig, 2009]

Answers **(DID YOU TRY FIRST?):**

1. ΔV = 0.096 km/s
2. ΔV = 0.491 km/s
3. ΔV = 0.587 km/s
4. ΔV1+ΔV2=ΔV3
5. ΔV = 0.159 km/s

Interplanetary transfers

1st-order computation of interplanetary trajectories:

- orbits of planets are assumed to be circular
- orbits of planets are assumed to be coplanar
- a transfer is split into 3 successive unperturbed 2-body problems ("patched conics"):

  1. a section where the gravity of the departure planet is dominant
  2. a section where the gravity of the Sun is dominant
  3. a section where the gravity of the target planet is dominant

- Sphere of Influence (Earth: ~ 10^6 km)

Compare the dimension of the Sphere of Influence (SoI) with an Astronomical Unit: -> satellite will spend far majority of flight time in heliocentric phase. Patched comics approach is good to distinguish between "local" (i.e. around planets) and "global" (i.e. around Sun) phases of flight, and get 1st-order solutions for the satellite motions (using relations for standard Kepler orbits).

Interplanetary transfers (cnt'd)

Transfers: 3 successive unperturbed 2-body problems

Here:

- only central (heliocentric) part is considered
- Hohmann transfer orbits
- ΔV at departure planet and target planet is mismatch in velocity, not the actual maneuver

Planetary orbits

planet	mean distance from Sun [AU]	eccentricity [-]	inclination [°]	sidereal period [yr]	synodic period [yr]	mean orbital velocity [km/s]
Mercury	0.387	0.206	7.00	0.241	0.317	47.78
Venus	0.723	0.007	3.39	0.615	1.599	35.03
Earth	1.000	0.017	0.00	1.000	-	29.78
Mars	1.524	0.093	1.85	1.881	2.135	24.13
Jupiter	5.203	0.049	1.30	11.862	1.092	13.06
Saturn	9.555	0.056	2.49	29.458	1.035	9.64
Uranus	19.218	0.047	0.77	84.014	1.012	6.81
Neptune	30.110	0.009	1.77	164.79	1.006	5.43
Pluto	39.440	0.248	17.17	248.5	1.004	4.74

The same transfer problem, but now on the scale of the solar system. This table gives some relevant data on the orbits of the planets.

Planetary orbits (cnt'd)

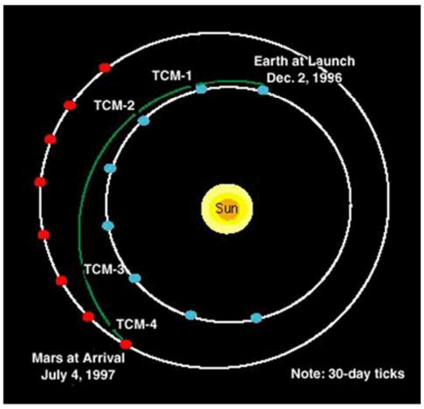
Some notes on table on previous sheet:

- Pluto is no longer an official planet since 2006
- 1 Astronomical Unit (AU) is the mean distance between the Earth and the Sun (149.6×10^6 km)
- eccentricities and inclinations are small for most planets
- $1/T_{synodic} = abs(1/T_{planet} - 1/T_{Earth})$, the period that a specific relative geometry repeats
- the orbital velocities can be verified with equations for Kepler orbits, with Sun as primary body ($\mu_{Sun} = 1.3271 \times 10^{11}$ km^3/s^2)

Interplanetary transfers: example

Hohmann transfers: also on interplanetary scale

Mars Pathfinder:



[Oarval, 2009]

[NASA, 2009]

As with Hohmann transfers between orbits around the Earth, a Hohmann transfer between orbits (here: between two celestial bodies) is a minimum-energy transfer. Its apocenter and pericenter are tangential to the departure and arrival orbits, and it covers 180° in true anomaly.

Interplanetary transfers: Hohmann

Hohmann transfers on an interplanetary scale:

- Minimum energy-transfers
- Typically based on instantaneous velocity changes (ΔV's) at begin and end

- Semi-major axis: $$a_{transfer} = (r_{planet1} + r_{planet2})/2$$

- Transfer time: $$T_{transfer} = \pi\sqrt{\frac{a_{transfer}^3}{\mu_{Sun}}}$$

- Faster and slower orbits are possible (so-called Type-I and Type-II), but these will cost more energy
- Alternatives: use planetary (aero)gravity-assists, or use alternative propulsion techniques (low-thrust, solar sailing, ….)

Interplanetary transfers: Hohmann (cnt'd)

Example: Hohmann transfer from Earth to Mars (all velocity values are heliocentric)

- semi-major axis = 1.262 AU = 188.8×10^6 km
- eccentricity = 0.208
- transfer time = 22371805 sec = 258.9 days
- $V_{pericenter}$ = 32.73 km/s
- $V_{apocenter}$ = 21.48 km/s
- V_{Earth} = 29.78 km/s (cf. sheet "Planetary orbits")
- V_{Mars} = 24.13 km/s (idem)
- $V_{\infty,Earth}$ = 2.95 km/s
- $V_{\infty,Mars}$ = 2.65 km/s

(to be continued)

The semi-major axis is the average of the pericenter radius and the apocenter radius.

The velocities in pericenter and apocenter are computed with the standard vis-viva equation for the velocity in an orbit: $\frac{1}{2} V^2 - \mu/r = - \mu/(2a)$.

The velocities for Earth and Mars itself are circular velocities. The excess velocities $V\infty$ are the (absolute) difference between the velocity in the ellipse and the corresponding circular velocity of the relevant celestial body. So: heliocentric velocity must be increased at Earth by 2.95 km/s, in order to "take the wider swing to Mars", and be lowered at Mars by 2.65 km/s in order to catch up with the circular velocity of Mars itself.

Questions

Consider a Hohmann transfer from Earth (at 1 AU) to Jupiter (at 5.2 AU).

What is:

1. the semi-major axis of the transfer orbit?
2. the eccentricity of the transfer orbit?
3. the pericenter velocity in this transfer orbit?
4. the apocenter velocity in this transfer orbit?
5. the travel time?

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

$\mu_{Sun} = 1.3271 \times 10^{11}$ km^3/s^2; 1 AU = 149.6×10^6 km

Answers: **(DID YOU TRY??)**

1. a = 3.1 AU = 463.8×10^6 km
2. e = 0.677
3. V_p = 38.575 km/s
4. V_a = 7.418 km/s
5. T = 86137877 s = 997 days = 2.73 yrs

Questions

Consider a transfer from Earth (at 1 AU) to Jupiter (at 5.2 AU). Assume the spacecraft leaves the Earth orbit in a direction tangential to the orbit of the Earth itself, with an excess velocity ΔV equal to 11 km/s. (hint: this is not a Hohmann transfer orbit)

What is:

1. the velocity of the Earth?
2. the semi-major axis of the transfer orbit?
3. the eccentricity of the transfer orbit?
4. the value for the true anomaly θ when the vehicle leaves the Earth vicinity?
5. the value of θ when the vehicle arrives at the target planet?
6. the travel time?

Answers: see footnotes below **(BUT TRY YOURSELF FIRST!!)**

$\mu_{Sun} = 1.3271 \times 10^{11}$ km^3/s^2; 1 AU = 149.6×10^6 km

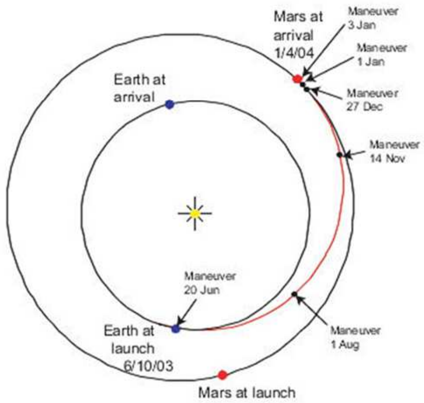
Answers: **(DID YOU TRY??)**

1. V_{Earth} = 29.784 km/s
2. a = 1.197×10^9 km = 8.00 AU
3. e = 0.875
4. θ = 0°
5. θ = 136.95°
6. T = 40618428 s = 470.12 days = 1.29 yrs

Interplanetary transfers: examples

maneuvers on an interplanetary scale

Mars Exploration Rover "Spirit"



[NASA, 2009]

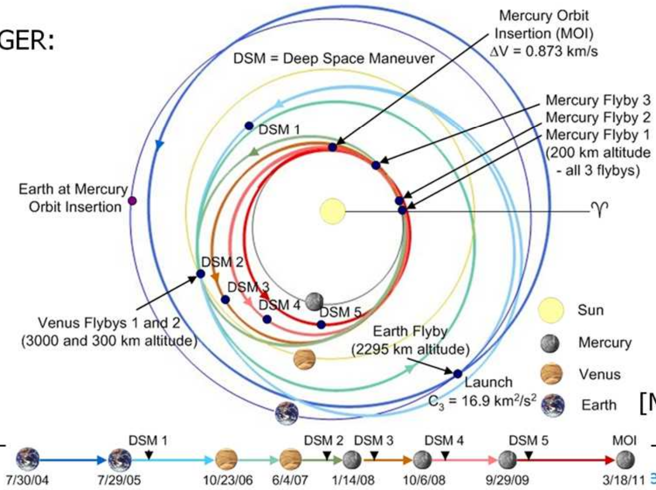
The MER "Spirit" was launched on June 10, 2003, for a Martian roving mission planned to take 90 sols (i.e. 90 Martian days, 24h 37m 22s each). It is accompanied by a twin rover "Opportunity". Both are still operating in Summer 2009.

The orbit from Earth to Mars flown by "Spirit" is a so-called type-I orbit: shorter (faster) than a minimum-energy Hohmann orbit.

Interplanetary transfers: examples

maneuvers on an interplanetary scale

MESSENGER:



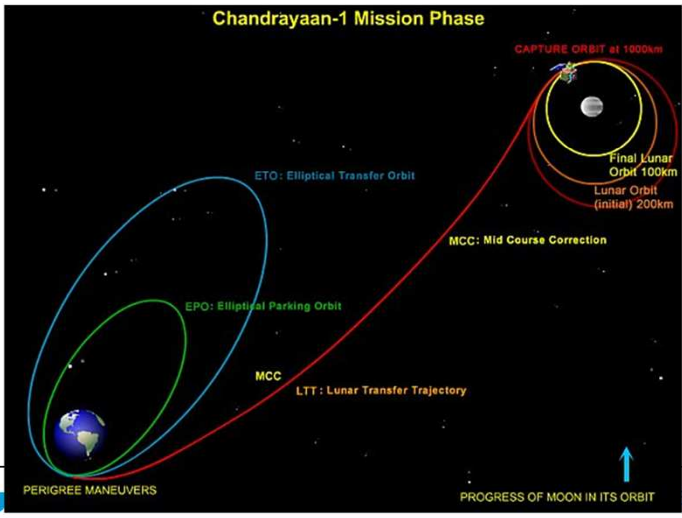
[JHU/APL, 2009]

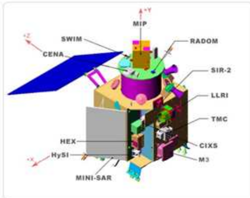
[Mercurytoday, 2009]

The MErcury Surface, Space ENvironment, GEochemistry, and Ranging (MESSENGER) mission, launched on August 3, 2004.

Interplanetary transfers: examples

maneuvers on an interplanetary scale





[ISRO, 2009]

Chandrayaan-1

The Indian Moon-mission Charndrayaan-1, launched on October 22, 2008.

ΔV budget

Why?

- structured way to identify and quantify all maneuvers
- combine all maneuvers in single parameter
- translate to required propellant mass
- evaluate various options mission scenario

ΔV budget

	LEO	GEO	interplanetary
launch	typically not included in budget		
orbit transfer (1st burn)	initiation of Hohmann orbit	initiation of Hohmann orbit	initiation of Hohmann orbit to target planet
orbit transfer (2nd burn)	end of Hohmann orbit	end of Hohmann orbit	no
altitude maintenance	mission dependent (drag, …)	no	no
N/S station keeping	no	51.38 m/s/yr	no
E/W station keeping	no	1.7 sin(2(λ-75)) m/s/yr	no
rephasing, rendezvous	mission dependent	mission dependent	mission dependent
node, plane change	mission dependent	no	mission dependent
disposal	initiation of Hohmann orbit to e.g. 200 km	Hohmann orbit to graveyard orbit (GEO + 300): ~ 7.4 m/s	no
orbit insertion	no	no	mission dependent
total	summ of the above		